Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover Lecture 12

Topic: Quantum magnetism: Oshikawa's extension of LSM theorem

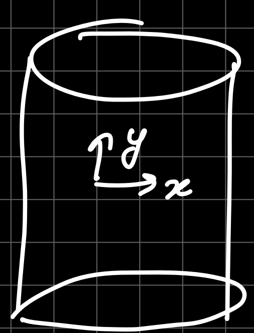
Oshikawa's extension of LSM

filling $\nu = \frac{p}{q}$ $\exists$ q - low lying states

doubly deg. GS

$$\mathcal{H} = \mathcal{H}_{hopping} + \mathcal{H}_{int}$$
$$= -t \sum (b_i^+ b_j + h.c.) + V(\{n\})$$

work in $L = 2$ & on a cylinder



$L_x \times L_y$ = size

insert flux ϕ

change $\phi : 0 \to 2\pi$

$b \to e^{i\theta} b$

leads to momentum change

$$\frac{\partial \phi}{\partial t} \sim \int (\nabla \times E)\, ds$$
$$\sim \oint \vec{E} \cdot \vec{dl}$$
$$\Delta p \sim \int E\, dt \sim \Delta \phi$$

$$(b_r^\dagger b_{r'} + h.c.) \to (b_r^\dagger b_{r'} e^{i a_{rr'}} + h.c.)$$

$a =$ constant a f^n of space

We choose

$$a_{r,r+\hat{x}} = \frac{\phi}{L_x} \qquad a_{r,r+\hat{y}} = 0$$

$$\mathcal{H}(\phi) = -t \sum_r b_r^\dagger b_{r+\hat{x}} e^{i\phi/L_x} + h.c.)$$

$$- t \sum_r (b_r^\dagger b_{r+\hat{y}} + h.c.) + V\{n_r\}$$

Increase ϕ from 0 to 2π adiabatically.

$$U = \exp\left(\frac{2\pi i}{L_x} \sum_r x\, n_{\vec{r}}\right)$$

$$U^\dagger \mathcal{H}(\phi = 2\pi)\, U = \mathcal{H}(\phi = 0)$$

$$U^\dagger b_r^\dagger U = e^{\frac{2\pi i x}{L}} b_r^\dagger \qquad U^\dagger b_{r+\hat{x}} U = e^{-\frac{2\pi i (x+1)}{L_x}} b_{r+\hat{x}}$$

$$\mathcal{H}(\phi = 0)\,|\psi_0\rangle = E_0 |\psi_0\rangle$$

$$T_x |\psi_0'\rangle = e^{iP_0} |\psi_0\rangle$$

Map spin $1/2$ chains to hardcore bosons

$$S^+ \sim b^\dagger, \quad S^- \sim b \qquad S^z = b^\dagger b - \tfrac{1}{2}, \qquad b^\dagger b \in (0,1)$$

$\langle S^z \rangle = 0 \to$ filling is $\frac{1}{2} \to$ at least 2 low lying states

$$\vec{S} = f^\dagger \frac{\vec{\sigma}}{2} f$$

* TR + translation invariance $\}$ $\Rightarrow$ believe that $\exists$ one more state

<u>Kramer's theorem</u> :- τ

$[\tau, \mathcal{H}] = 0 \qquad \tau^2 = -1 \qquad \tau^+ i \tau = (-i)$

$\mathcal{H}|\psi\rangle = E|\psi\rangle \Rightarrow \tau|\psi_1\rangle = |\psi_2\rangle$

if $|\psi_2\rangle = e^{i\theta}|\psi_1\rangle \Rightarrow \tau|\psi_2\rangle = e^{-i\theta}|\psi_2\rangle$

$= \underline{|\psi_1\rangle}$

$T|\psi_1\rangle = |\psi_2\rangle \Rightarrow$

$\tau^2|\psi_2\rangle = \tau|\psi_1\rangle$

$-|\psi_2\rangle = \tau|\psi_1\rangle = |\underline{\psi_2}\rangle$ $\rightarrow$ contradiction.